Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
International Journal of Thermal Sciences 42 (2003) 821–828
International Journal of Thermal Sciences
www.elsevier.com/locate/ijts

Laminar convective heat transfer with twisted tape inserts in a tube

P.K. Sarma [a,*], T. Subramanyam [b], P.S. Kishore [a], V. Dharma Rao [c], Sadik Kakac [d]

[a] Department of Mechanical Engineering, College of Engineering, GITAM, Rushikonda, Visakhapatnam 530 045, India
[b] Department of Mechanical Engineering, College of Engineering, Andhra University, Visakhapatnam 530 003, India
[c] Department of Chemical Engineering, College of Engineering, Andhra University, Visakhapatnam 530 003, India
[d] Department of Mechanical Engineering, College of Engineering, University of Miami, Coral Gables, FL 33124, USA

Received 4 February 2002; accepted 24 April 2002

Abstract

A new method is postulated to predict heat transfer coefficients with twisted tape inserts in a tube, in which the wall shear and the temperature gradients are properly modified through friction coefficient correlation leading to heat transfer augmentation from the tube wall. The eddy diffusivity expression of van Driest is modified to respond to the case of the internal flows in a tube with twisted tapes for ranges of Reynolds number corresponding to the laminar flow in tubes. The predictions from the present theory are compared with some correlations available in literature for twisted tape inserts. The present theoretical results are rendered in the form of a correlation equation.

Keywords: Laminar convective heat transfer; Twisted tape inserts; Eddy diffusivity; Van Driest expression; Correlations

1. Introduction

Augmentation of convective heat transfer in internal flows with tape inserts in tubes is a well-acclaimed technique employed in industrial practices. The state-of-the-art on passive and active heat transfer techniques is excellently reviewed by Bergles [1]. Correlations are now available both for laminar and turbulent regimes in literature [2–18]. Very recently, Manglik and Bergles [2,3] have presented comprehensively their analyses in parts I and II. Their studies are devoted respectively to the heat transfer and pressure drop correlations for laminar and turbulent regimes with twisted tapes. It can be observed from their study that the development of the correlations is based on a variety of effects included through dimensionless parameters. The five factors listed by them and included in their study are:

- increase in flow velocity due to portioning of the tube;
- decrease in hydraulic diameter leading to increase in heat transfer coefficient;
- increase in flow path due to helical path of the particle following the configuration of the twisted tape;
- increase in the convective heat transfer due to secondary motion generated by the presence of the tape;
- the fin contribution if the tape insert is in good thermal contact with the wall of the tube.

According to them the sum total of all these effects would enhance both heat transfer and pressure drop even at low Reynolds numbers, such as *Re* less than 2300. Thus, they introduced these concepts to develop correlations. However, a major influencing factor leading to enhancement can be attributed to the intensive cross mixing of the fluid due to centrifugal motion of the particles due to the tape twist. Consequently, the turbulent/eddy viscous forces generated are quite intensive in the core in addition to the molecular viscous forces predominantly existing only in the proximity of the wall region. Thus, the presence of the tape physically induces effective cross mixing even at magnitudes of Reynolds numbers that correspond to laminar regime in tubes. In an earlier paper Sarma et al. [13] modified the eddy diffusivity expression of van Driest for turbulent regime in which the universal constant K is found to be a function of pitch-to-diameter ratio of the given configuration of the tape insert. The present article is an extension of this concept to the laminar regime. The theory developed is compared with

* Corresponding author.
E-mail addresses: sarmapullela@hotmail.com (P.K. Sarma), tsmanyam77@hotmail.com (T. Subramanyam), vdharmarao@yahoo.com (V.D. Rao), skakac@miami.edu (S. Kakac).

1290-0729/$ – see front matter
doi:10.1016/S1290-0729(03)00055-3

Nomenclature

A^+	constant in van Driest expression, Eq. (1)	
C_p	specific heat at constant pressure	$J\cdot kg^{-1}\cdot K^{-1}$
D	inside diameter of the tube	m
f	friction coefficient	
H	helical pitch of the twisted tape	m
h	heat transfer coefficient	$W\cdot m^{-2}\cdot K^{-1}$
K	universal constant in the van Driest expression, Eq. (1)	
k	thermal conductivity	$W\cdot m^{-1}\cdot K^{-1}$
m	flow rate	$kg\cdot s^{-1}$
Nu	Nusselt number, $= hD/k$	
Pr	Prandtl number, $= \mu C_p/k$	
R	inside radius of the tube	m
R^+	dimensionless radius	
Re	Reynolds number, $= 4m/\pi D\mu$	
T	temperature	°C
T^+	dimensionless temperature	
u	velocity	$m\cdot s^{-1}$
u^*	shear velocity	$m\cdot s^{-1}$
u_m	mean velocity	$m\cdot s^{-1}$
u^+	dimensionless velocity	
y	distance measured normal to the tube wall	m
y^+	dimensionless distance	

Greek symbols

ρ	density	$kg\cdot m^{-3}$
μ	absolute viscosity	$kg\cdot m^{-1}\cdot s^{-1}$
τ	shear stress	$N\cdot m^{-2}$
ν	kinematic viscosity	$m^2\cdot s^{-1}$
ε_m	turbulent eddy viscosity	$m^2\cdot s^{-1}$
ε_h	thermal eddy diffusivity	$m^2\cdot s^{-1}$

Subscripts

B	mean bulk
c	centerline
w	wall

the heat transfer data of Lecjaks et al. [14], correlations of Hong and Bergles [10], Agarwal and Rajarao [12]. Thus, the satisfactory agreement between their theory and the experimental data establishes the utility of modified van Driest's expression as a tool in the convective heat transfer study with the tape inserts as turbulence promoters in the laminar regime as well.

This article outlines the possible application of the van Driest's mixing length theory to other possible types of flows generated by some extraneous objects in the internal flow systems. In the present study it is the flow with tape inserts.

In this context though the flow Reynolds numbers correspond to the laminar regime, enhancement of turbulence leading to higher rates of heat dissipation from the wall due to the presence of tape. The flow might be construed as the one belonging to psuedo-laminar regime and the usage of the term laminar is a misnomer. Thus, the physics of the problem is closer to the turbulent flows even when $Re < 2300$. The word laminar in the strictest sense is to be defined in terms of the conditions of hydrodynamics of the flow in the plain tube without tape insertions. Specifically the momentum characteristics play significant role.

Thus, the present article is a modest attempt to extrapolate the mixing length theory to cases such as tape insertions in a tube for a range of Re corresponding to laminar regime for internal flows in pipes.

2. Formulation and solution

The following assumptions are employed in formulating the problem:

- The parameters as stated by Manglik and Bergles [2,3] leading to enhancement of heat transfer are significant contributing to heat transfer and pressure drop in laminar internal flows with tape inserts. Nevertheless, the influence of all these parameters except the fin effects can be indirectly taken care by K in van Driest expression for turbulent eddy diffusivity by considering $K = K(H/D, Re)$. The presence of tape induces cross-mixing and tangential component to the flow and hence the combined effect is to enhance the eddy shear in the flow.
- There are several correlations existing in the literature as depicted in Fig. 1. It is observed that these correlations exhibit considerable deviation with one another. For this reason it is preferred to use the recent friction factor correlation of Kishore [11] for laminar regime with twisted tape inserts for the high Prandtl number fluid Turbinol XT-32 in the evaluation of wall shear characteristics with inserts. The correlation is as follows:

$$f_{\text{tape}} = 1.5\left[1 + \frac{D}{H}\right]^{3.37} Re^{-0.6} \tag{1}$$

  Equation of Kishore [11] holds good for the range $200 < Pr < 400$, $2.5 < [H/D] < 20$.
- The presence of the tape is ignored treating the flow as the one corresponding to an equivalent turbulent flow in a tube in which the wall friction coefficient being prescribed by Eq. (1) for any given value of $Re < 3000$.
- The eddy diffusivity equation of van Driest holds good and the universal constant K in the equation can be a function of $[H/D, Re]$ for flow with twisted tape inserts.

$$\frac{\varepsilon_m}{\nu} = \left[Ky^+\left\{1 - \exp\left(-y^+/A^+\right)\right\}\right]^2 \left|\frac{du^+}{dy^+}\right| \tag{2}$$

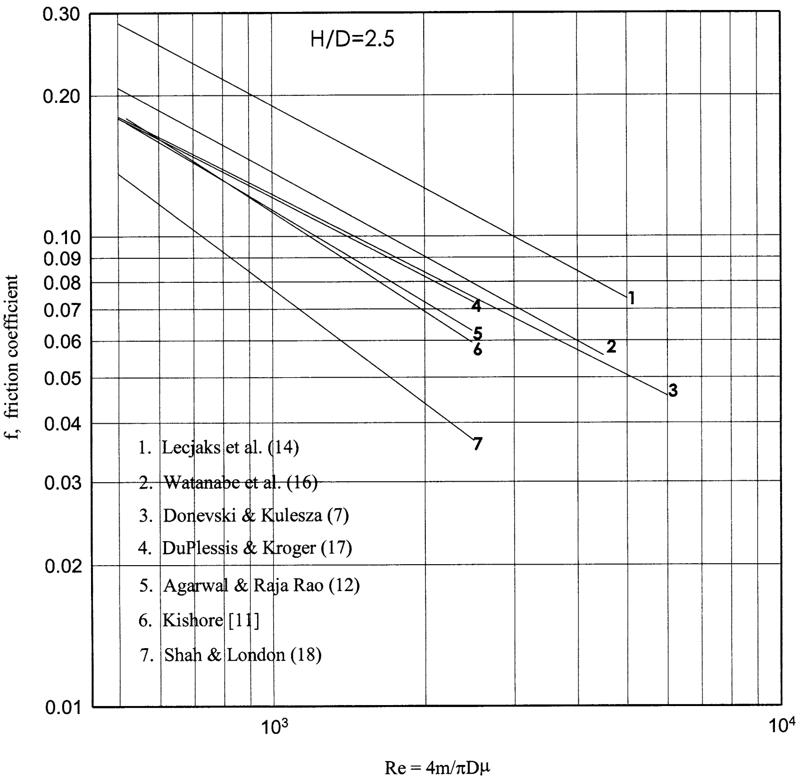

Fig. 1. Comparison of friction coefficients with various analysis.

where $A^+ = 26$ as suggested by van Driest for turbulent flow. K is a function of $[H/D]$ which is to be determined.

Variation of either K or A^+ will have its influence on the derivatives of velocity and temperature. Eq. (2) can also be applied to the laminar flows with K being considered as a function of the parameters Re and tape twist ratio $[H/D]$.

- Since the flow is considered as an equivalent through a circular tube the shear stress variation is assumed linear across the tube, i.e.,

$$\frac{\tau}{\tau_w} = \left[1 - \frac{y^+}{R^+}\right] \tag{3}$$

where

$$y^+ = \frac{yu^*}{\nu}$$

$$R^+ = \frac{Ru^*}{\nu}, \qquad u^* = \sqrt{\frac{\tau_w}{\rho}}$$

$$\tau_w = 0.5 f_{\text{tape}} \rho u_m^2$$

- The swirl flow is assumed to be thermally developed and turbulent/eddy conduction is the dominant mode. For fluids with $Pr > 1$ it can be considered that $\varepsilon_m = \varepsilon_h$.
- The physical properties are independent of temperature variation across the tube and hence the physical property variation is ignored.

These assumptions would enable us to formulate the problem for investigating the effect of swirl induced on temperature and velocity fields. Since an equivalent swirl flow is considered in a circular tube from the assumption of linear shear distribution it can be shown that

$$\frac{du^+}{dy^+} = \left[\frac{1 - (y^+/R^+)}{1 + (\varepsilon_m/\nu)}\right] \tag{4}$$

The boundary condition is at $y^+ = 0$, $u^+ = 0$.

Combining Eq. (4) with the expression for eddy diffusivity, i.e., Eq. (2) it can be shown that

$$\frac{du^+}{dy^+} = \left(-1 + \left[1 + 4K^2 y^{+2}\left\{1 - \frac{y^+}{R^+}\right\} \times \left[1 - \exp\left(-\frac{y^+}{A^+}\right)\right]^2\right]^{1/2}\right) \times \left(2\left[Ky^+\left\{1 - \exp\left(-\frac{y^+}{A^+}\right)\right\}\right]^2\right)^{-1} \tag{5}$$

where

$$R^+ = \frac{1}{2}\left[\frac{Du_m}{\nu}\right]\sqrt{\frac{f_{\text{tape}}}{2}} = \frac{Re}{2}\sqrt{\frac{f_{\text{tape}}}{2}} \tag{6}$$

$$Re = \frac{4m}{\pi D\mu}$$

3. Evaluation of K in van Driest expression for swirl flow

For swirl flow generated due to tapes the value of K in van Driest's mixing length expression is iterated as per the procedure outlined.

The steps are as follows:

(1) Prescribe $Re, H/D$ and K. Calculate the value of the average friction coefficient f_{tape} from Eq. (1).
(2) Calculate R^+ from Eq. (6) making use of Eq. (1) for a prescribed value of Re which should be less than 2300.
(3) Solve differential equation (5) to establish $u^+ = u^+(y^+)$. Hence, calculate

$$Re = \left[\frac{4m}{\pi D\mu}\right] = 4\int_0^{R^+} u^+\left[1 - \frac{y^+}{R^+}\right]\mathrm{d}y^+ \tag{7}$$

(4) Check whether Re from step (3) is equal to the prescribed value in step (1). If the agreement is noted within limits of accuracy, the value of K in step (1) is taken as the correct value. Or else proper iteration on K is repeated till Re of step (1) equals value of Re of step (3).

Such computations are performed with the aid of friction coefficient Eq. (1) for the range $300 < Re < 3000$. Thus, the variation of K in van Driest expression, i.e., Eq. (1) is shown plotted in Fig. 2. As can be seen from the plot for low values of $[H/D]$ the variation in K is more significant. However for $[H/D] > 10$, the variation of K with respect to the Reynolds number is marginal. K is approximated by the expression as follows:

$$K = 1.767Re^{-0.232}\left[1 + \pi\left(\frac{D}{H}\right)\right]^{2.2} \tag{8}$$

Eq. (8) is accomplished by applying regression to the values of K from the theory covering wide range of $[H/D]$. The variation of eddy diffusivity is shown in Fig. 3 for various values of $[H/D]$ for a given $Re = 500$. It can be seen that as per the model as $[H/D]$ decreases the intensity of eddy viscosity increases due to the enhancement of radial component of velocity of the particles. Such an observation is consistent with the model, viz., that insertion of the tape even at low Reynolds number induces intensive cross mixing leading to enhancement of turbulent shear. Thus, the inference from Figs. 2 and 3 is that the velocity gradients at the wall will be profoundly influenced at the wall due to the presence of the tape. This result is extended to predict the heat transfer coefficient for a given tape insert for the range of $300 < Re < 3000$.

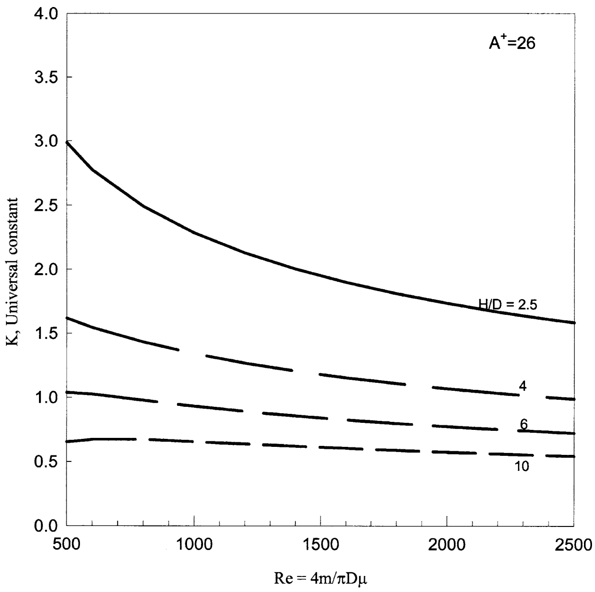

Fig. 2. Variation of K with Re for $2.5 < [H/D] < 10$.

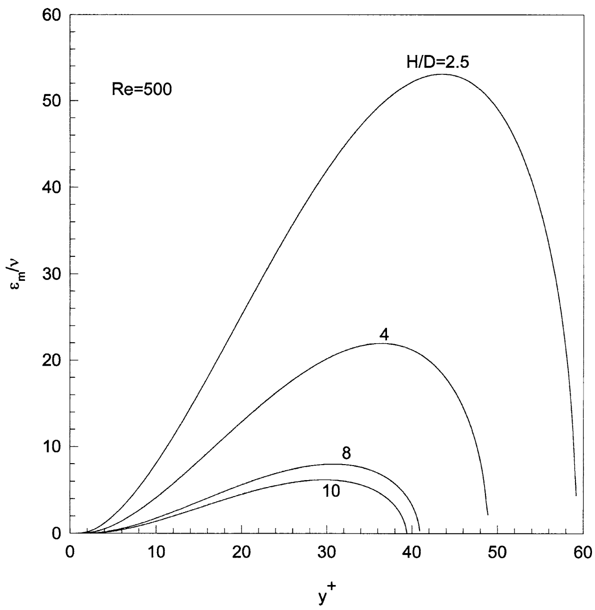

Fig. 3. Effect of tape twist ratio on eddy diffusivity.

4. Estimation of heat transfer coefficients

As per the assumptions the temperature field across the tube can be obtained from the following dimensionless equation:

$$\frac{\mathrm{d}}{\mathrm{d}y^+}\left[\left(1 + \frac{\varepsilon_m}{\nu}Pr\right)\frac{\mathrm{d}T^+}{\mathrm{d}y^+}\right] = 0 \tag{9}$$

where

$$T^+ = \frac{T_w - T}{T_w - T_c}$$

The boundary conditions are

$$T^+ = 0 \quad \text{at } y^+ = 0$$
$$T^+ = 1 \quad \text{at } y^+ = R^+ \tag{10}$$

The heat transfer coefficient can be obtained by definition from

$$h = -\frac{k}{(T_w - T_B)} \frac{dT}{dy}\bigg|_{y=0} \tag{11}$$

Or in dimensionless form

$$Nu_m = \frac{hD}{k} = 2R^+ \frac{dT^+}{dy^+}\bigg|_{y^+=0} \left[\frac{T_w - T_c}{T_w - T_B}\right] \tag{12}$$

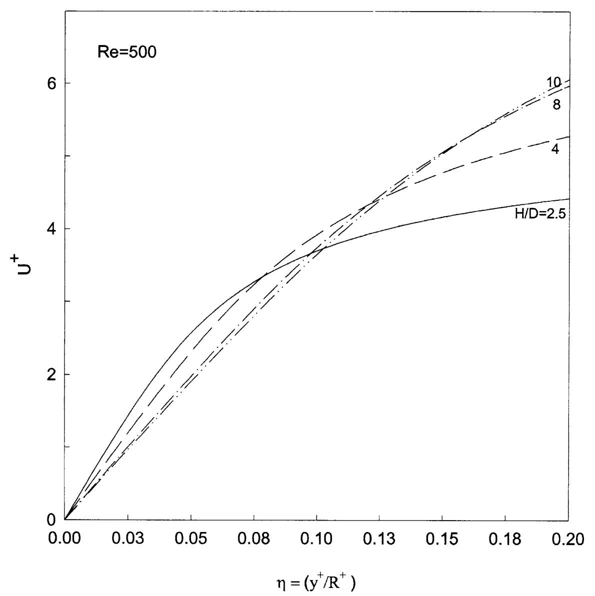

Fig. 4. Velocity profile in the wall region.

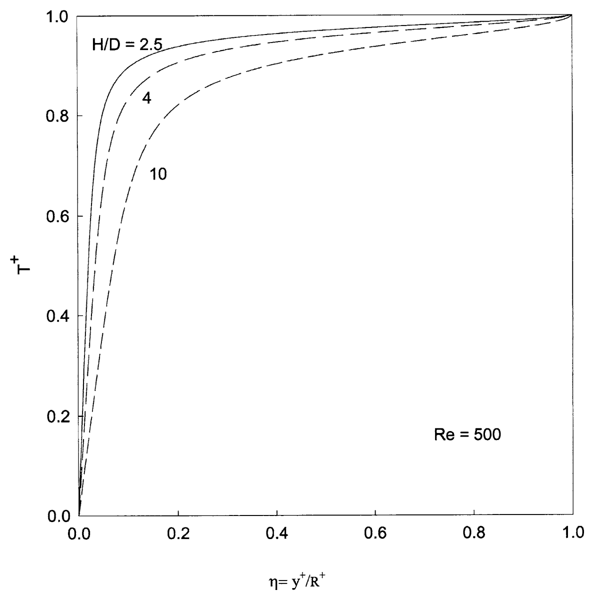

Fig. 5. Effect of tape twist ratio on temperature profile.

The temperature ratio term in Eq. (12) can be obtained from the velocity and temperature profiles as follows

$$\frac{T_w - T_c}{T_w - T_B} = \frac{\int_0^{R^+} u^+[1 - y^+/R^+]\,dy^+}{\int_0^{R^+} u^+T^+[1 - y^+/R^+]\,dy^+} \tag{13}$$

The results of the analysis are further discussed to bring out the influence of $[H/D]$ on velocity and temperature profiles through K in the van Driest expression in Figs. 4 and 5. It can be observed from Fig. 4 that a decrease in the value of $[H/D]$ makes the profile flatter and uniform in the core region. Further, the velocity gradients become steeper in the wall region indicating increase in the wall resistance, which in turn leads to enhancement of friction coefficient. The influence of $[H/D]$ on the temperature gradients is also found to be quite profound as can be seen from Fig. 5. It can be inferred that the heat flux dissipated from the wall increases substantially with the decrease in the value of $[H/D]$. Thus, as per the present model it can be seen that decrease in $[H/D]$ indirectly supports the reasoning that increase in the tangential component of velocity due to tape insertion makes the sub layer thinner and hence temperature gradients are substantially increased leading to enhancement of heat transfer coefficients. Though the trends of the temperature and velocity profiles are in conformity with the physical conditions of flow with tape inserts further validation is taken up by comparing the theoretical predictions with some of the existing correlations in the literature.

5. Validation of the present theory

Relatively recent correlations of Hong and Bergles [10], Agarwal and Rajarao [12], and Lecjaks et al. [14] have been chosen for comparison of the present theory for $H/D = 4.8$

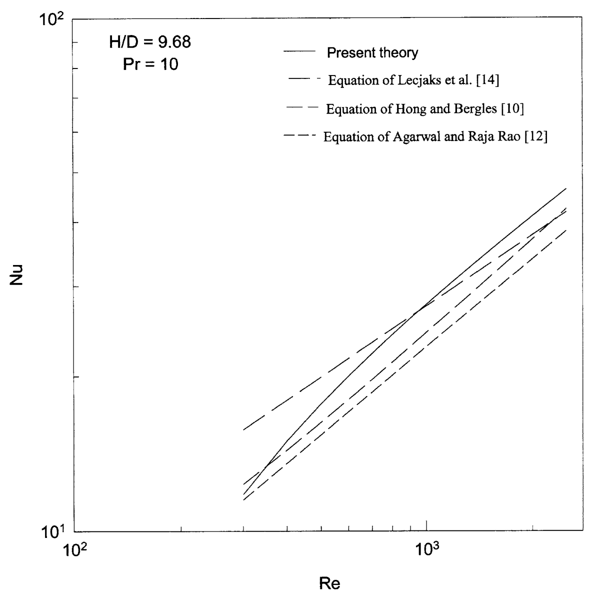

Fig. 6. Comparison of theory with other investigators.

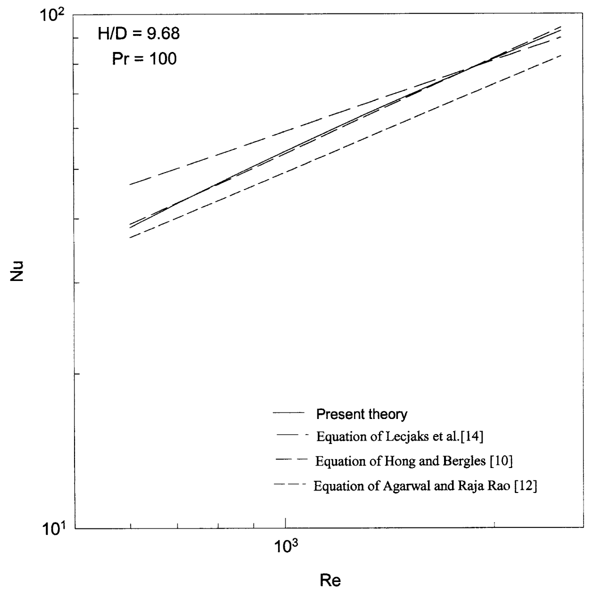

Fig. 7. Comparison of theory with other investigators.

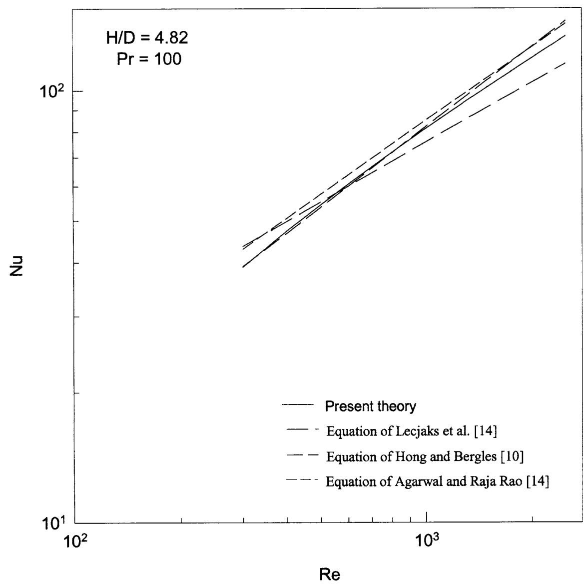

Fig. 9. Comparison of theory with other investigators.

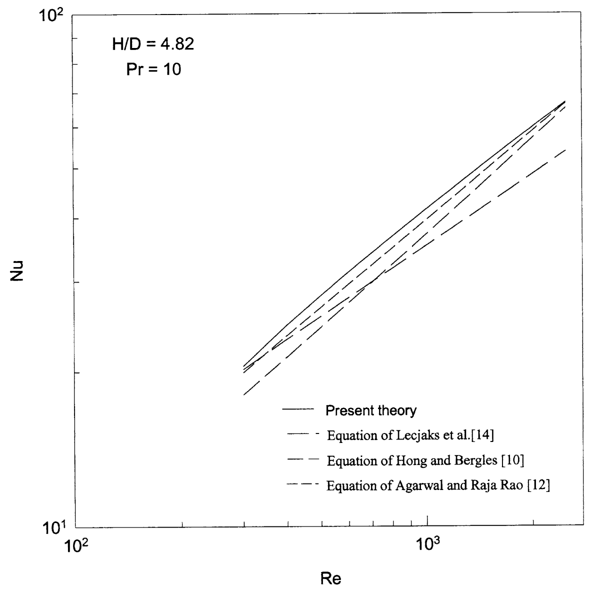

Fig. 8. Comparison of theory with other investigators.

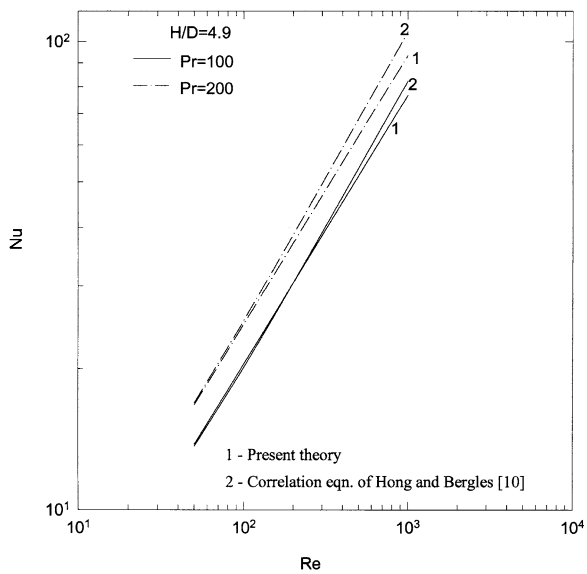

Fig. 10. Effect of Prandtl number—comparison.

and 9.68 and $Pr = 10$ and 100. The theoretical predictions are shown plotted in Figs. 6–10. Evidently, the results from the present analysis in these figures agree very satisfactorily with the correlations of these investigators. Further, the effect of Prandtl number is shown plotted in Fig. 10 together with the correlations of Hong and Bergles [10] for $Pr =$ 100–200. To avoid congestion of the lines and for the sake of clarity only comparison is limited to one investigator and obviously the effect of the Prandtl number is to increase the Nusselt number with the increase of Prandtl number. Since the agreement is found very satisfactory in all the Figs. 6–10, the results from theory are rendered into a dimensionless correlation by applying regression to nearly 250 points from the theory. The regression equation for the following ranges is as follows:

$$2.5 < H/D < 10 : 100 < Re < 3000 : 5 < Pr < 400$$
$$Nu_m = 0.2036 Re^{0.55} Pr^{0.3} [1 + D/H]^{4.12} \tag{14}$$

The correlation Eq. (14) from the theory is further compared with the data of Kishore [11] and Lecjaks et al. [14] for $7 < Pr < 400$ for different fluids such as water and the lubricant Turbinol XT-32 in Fig. 11. Though it can be seen that the theoretical line passes through the data points it can be refined as a mean line with a scatter of ±15% by regression analysis with the help of the following equation:

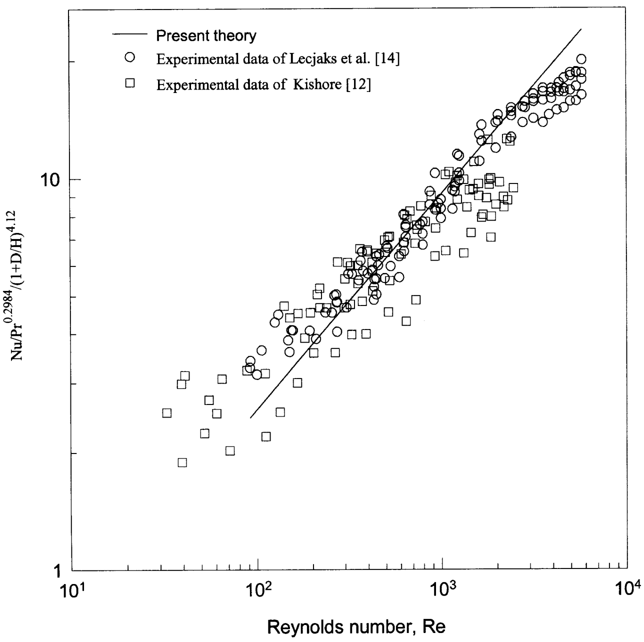

Fig. 11. Comparison of the theory with the experimental data.

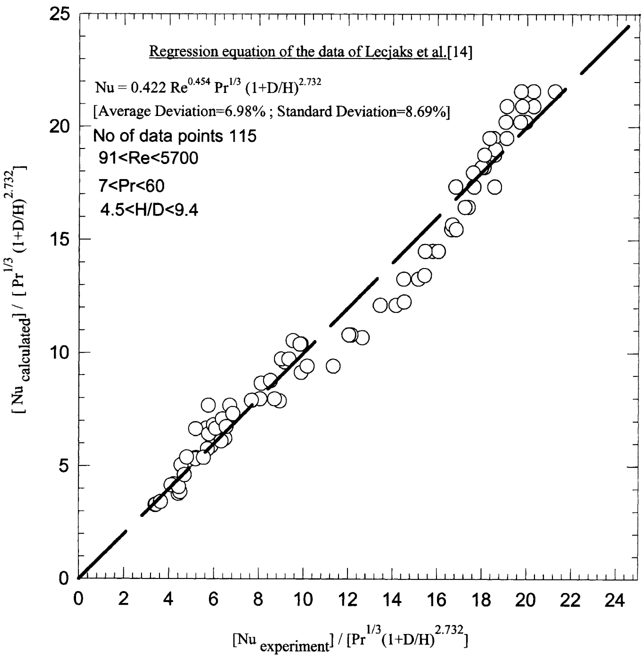

Fig. 12. Validation of the Regression equation—data of Lecjaks et al. [14].

$$Nu_m = 0.4031Re^{0.46}Pr^{1/3}[1 + D/H]^{2.781} \quad (15)$$

Eq. (15) is the correlation equation obtained by regression for the experimental data of Lecjaks et al. [14]. The data points scattered around the line with deviation not more than ±10% as can be seen in Fig. 12. From the aforesaid, it can be seen that according to the present model knowing the momentum characteristics one can workout the heat transfer characteristics with reasonable accuracy with help of the mixing length concepts with a modifications to the constant K. Possibly different flow situations can be answered by an appropriate choice of K looking at the fact, for example, the same formulation yields the special case of pure classical laminar flow regime with $K = 0$, the asymptotic value of $Nu = 4.32$ and $K = 0.4$ gives results coinciding with the well established correlation of Dittus–Boelter equation for fully developed turbulent convection, viz., $Nu = 0.023Re^{0.8}Pr^{0.4}$. Consequently, it can be thought of intuitively for the case $0 < K < 0.4$ might respond to the transition from laminar to the turbulent regimes in pure convection. However, in the present case K as given by Eq. (8) responds to the case of special flow in tubes with tape insertions.

6. Conclusions

The following conclusions can be arrived from the study undertaken.

(1) The effects of various contributory parameters except the fin effects enhancing the heat transfer coefficients as described by Manglik and Bergles [2,3] can be directly assessed by taking K as a function of $[H/D]$ in the eddy diffusivity expression of van Driest. It might be possible that several classes of convective problems can be tackled through a modification of mixing length theory of van Driest.

(2) The laminar flow regime with tapes could be treated successfully for a wide range of parameters $3 < Pr < 400$, $2.5 < H/D < 10$ by considering that the role of eddy viscosity arising due to the presence of tape becomes more dominant over kinamatic viscosity. It can be seen from the analysis that eddy viscosity is implicitly defined by the tape configuration, i.e., $\varepsilon_m/\nu = F[H/D, y^+]$. Hence, the word laminar becomes a misnomer when a tape is inserted.

(3) The theory could be rendered into a correlation Eq. (14) yielding close agreement with the correlations of other investigators and hence it can be used as estimation tool to predict enhancement of heat transfer coefficients.

Acknowledgement

The authors thank U.G.C. & D.S.T. New Delhi and NSF, Washington for the assistance.

References

[1] A.E. Bergles, Techniques to augment heat transfer, in: W.M. Rohsnow, et al. (Eds.), Hand Book of Heat Transfer Applications, 2nd Edition, McGraw-Hill, New York, 1985.

[2] R.M. Manglik, A.E. Bergles, Heat transfer and pressure drop correlations for twisted tape inserts in isothermal tubes: Part 1—Laminar flows, J. Heat Trans. T. ASME 115 (1993) 881–889.

[3] R.M. Manglik, A.E. Bergles, Heat transfer and pressure drop correlations for twisted tape inserts in isothermal tubes: Part II—Transition and turbulent flows, J. Heat Trans. T. ASME 115 (1993) 890–896.

[4] E. Smithberg, F. Landis, Friction and forced convection heat transfer characteristics in tubes with twisted-tape swirl generators, J. Heat Trans. T. ASME 86 (1964) 39–49.

[5] R.F. Lopina, A.E. Bergles, Heat transfer and pressure drop in tape-generated swirl flow of single phase water, J. Heat Trans. T. ASME 91 (1969) 434–442.

[6] R. Thorsen, F. Landis, Friction and heat transfer characteristics in turbulent swirl flow subjected to large transverse temperature gradients, J. Heat Trans. T. ASME 90 (1968) 87–98.

[7] B. Donevski, J. Kulesza, Resistance coefficients for laminar and turbulent flow in swirling ducts, Arch. Termodyn. Splan. 9 (3) (1978) 497–506 (in Polish).

[8] A.W. Date, Prediction of fully developed flow in a tube containing a twisted tape, Internat. J. Heat Mass Transfer 17 (1974) 845–859.

[9] W.J. Marner, A.E. Bergles, Augmentation of highly viscous laminar heat transfer inside tubes with constant wall temperature, J. Expt. Ther. Fluid Sci. 2 (1989) 252–267.

[10] S.W. Hong, A.E. Bergles, Augmentation of laminar heat transfer in tubes by means of twisted tape inserts, J. Heat Trans. T. ASME 98 (1976) 251–256.

[11] P.S. Kishore, Experimental and theoretical studies of convective momentum and heat transfer in tubes with twisted tape inserts, Ph.D. Thesis, Andhra Univ. Visakhapatnam, India, 2001.

[12] S.K. Agarwal, M. Rajarao, Heat transfer augmentation for the flow of viscous liquid in circular tubes using twisted tape inserts, Internat. J. Heat Mass Transfer 39 (1996) 3547–3557.

[13] P.K. Sarma, T. Subramanyam, P.S. Kishore, V.D. Rao, S. Kakac, A new method to predict convective heat transfer in a tube with twisted tape inserts for turbulent flow, Internat. J. Thermal Sci. 41 (2002), in press.

[14] Z. Lecjaks, J. Machac Sir, Heat transfer to a Newtonian liquid flowing through a tube with an internal helical element, Internat. Chem. Engrg. Amer. Inst. Chem. Engrg. 27 (1987) 210–217.

[15] Z. Lecjaks, J. Machac Sir, Pressure loss in fluids flowing in pipes equipped with helical screws, Internat. Chem. Engrg. 27 (1987) 205–209.

[16] K. Watanabe, T. Taira, Y. Mori, Heat transfer augmentation in tubular flow by twisted tapes at high temperatures and optimum performance, Heat Transfer Japan. Res. 12 (3) (1983) 1–31.

[17] J.P. Du Plessis, D.G. Kroger, Friction factor prediction for fully developed laminar twisted tape flow, Internat. J. Heat Mass Transfer 27 (1984) 2095–2100.

[18] R.K. Shah, A.L. London, Laminar flow forced convection in ducts, in: T.F. Irvine Jr, J.P. Hartnett (Eds.), Adv. Heat Transfer, Suppl. 1 (1978) 379–381.